Chapter 2

Discrete Distributions

2.1 Random Variables of the Discrete Type

2.1-2 **(a)**

$$f(x) = \begin{cases} 0.6, & x = 1, \\ 0.3, & x = 5, \\ 0.1, & x = 10, \end{cases}$$

(b)

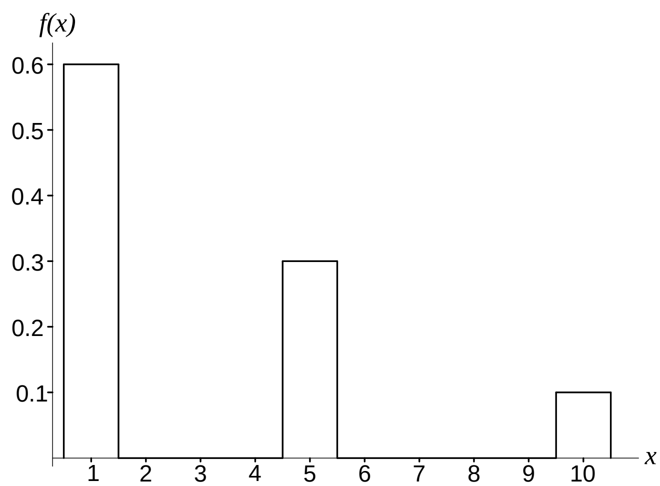

Figure 2.1–2: A Probability Histogram

2.1-4 **(a)** $f(x) = \dfrac{1}{10}\,,\ x = 0, 1, 2, \cdots, 10;$

(b)
$\mathcal{N}(\{0\})/150 = 11/150 = 0.073;$ $\mathcal{N}(\{5\})/150 = 13/150 = 0.087;$
$\mathcal{N}(\{1\})/150 = 14/150 = 0.093;$ $\mathcal{N}(\{6\})/150 = 22/150 = 0.147;$
$\mathcal{N}(\{2\})/150 = 13/150 = 0.087;$ $\mathcal{N}(\{7\})/150 = 16/150 = 0.107;$
$\mathcal{N}(\{3\})/150 = 12/150 = 0.080;$ $\mathcal{N}(\{8\})/150 = 18/150 = 0.120;$
$\mathcal{N}(\{4\})/150 = 16/150 = 0.107;$ $\mathcal{N}(\{9\})/150 = 15/150 = 0.100.$

(c)

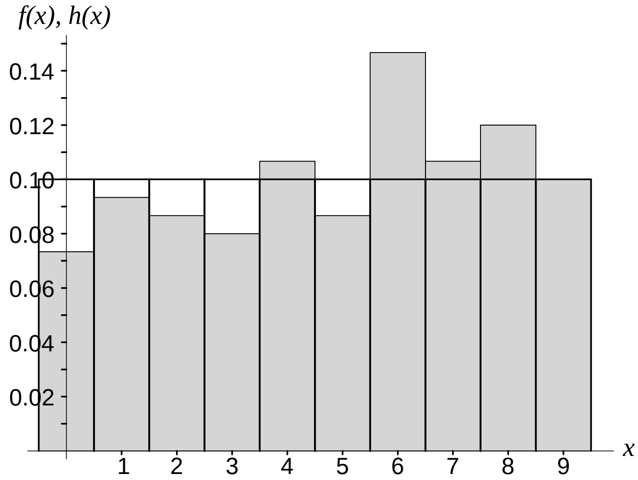

Figure 2.1–4: Michigan Daily Lottery Digits

2.1-6 **(a)** $f(x) = \dfrac{6 - |\,7 - x\,|}{36},\ x = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.$

(b)

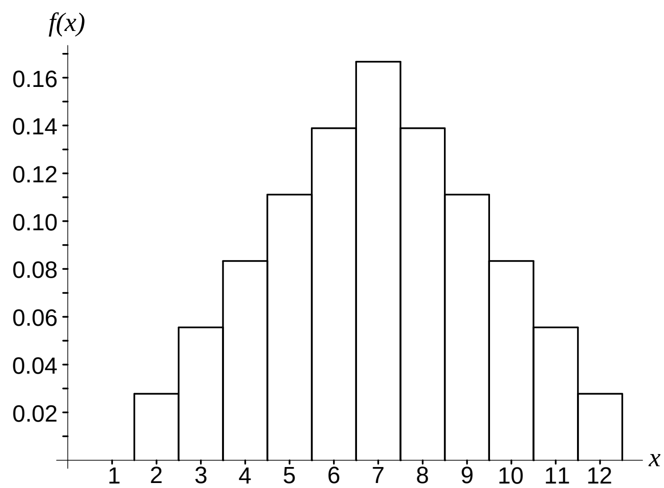

Figure 2.1–6: Probability histogram for the sum of a pair of dice

2.1-8 **(a)** The space of W is $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$.

$$P(W = 0) = P(X = 0, Y = 0) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}, \text{ assuming independence.}$$

$$P(W = 1) = P(X = 0, Y = 1) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}.$$

Continuing this, we see that $f(w) = P(W = w) = \dfrac{1}{12}, \quad w \in S$.

(b)

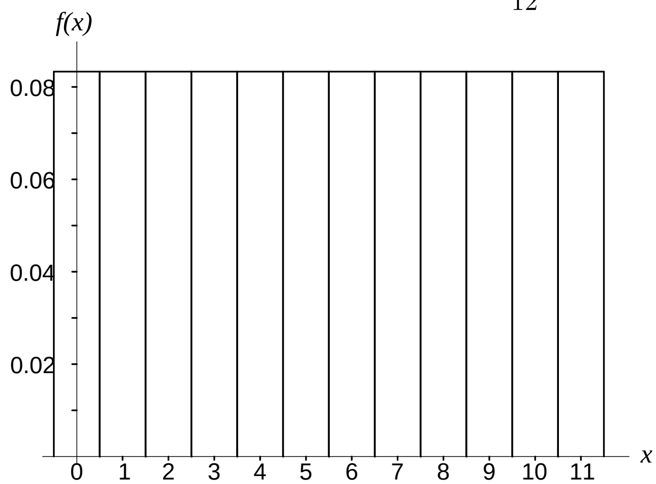

Figure 2.1–8: Probability histogram of sum of two special dice

2.1-10 **(a)** $$\frac{\binom{3}{1}\binom{47}{9}}{\binom{50}{10}} = \frac{39}{98};$$

(b) $$\sum_{x=0}^{1} \frac{\binom{3}{x}\binom{47}{10-x}}{\binom{50}{10}} = \frac{221}{245}.$$

2.1-12
$$OC(0.04) = \frac{\binom{1}{0}\binom{24}{5}}{\binom{25}{5}} + \frac{\binom{1}{1}\binom{24}{4}}{\binom{25}{5}} = 1.000;$$

$$OC(0.08) = \frac{\binom{2}{0}\binom{23}{5}}{\binom{25}{5}} + \frac{\binom{2}{1}\binom{23}{4}}{\binom{25}{5}} = 0.967;$$

$$OC(0.12) = \frac{\binom{3}{0}\binom{22}{5}}{\binom{25}{5}} + \frac{\binom{3}{1}\binom{22}{4}}{\binom{25}{5}} = 0.909;$$

$$OC(0.16) = \frac{\binom{4}{0}\binom{21}{5}}{\binom{25}{5}} + \frac{\binom{4}{1}\binom{21}{4}}{\binom{25}{5}} = 0.834.$$

2.1-14 $P(X \ge 1) = 1 - P(X = 0) = 1 - \dfrac{\binom{3}{0}\binom{17}{5}}{\binom{20}{5}} = 1 - \dfrac{91}{228} = \dfrac{137}{228} = 0.60.$

2.2 Mathematical Expectation

2.2-2

$$\begin{aligned} 1 &= \sum_{x=0}^{6} f(x) = \frac{9}{10} + c\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}\right) \\ c &= \frac{2}{49}; \\ E(\text{Payment}) &= \frac{2}{49}\left(1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{3} + 3 \cdot \frac{1}{4} + 4 \cdot \frac{1}{5} + 5 \cdot \frac{1}{6}\right) = \frac{71}{490} \text{ units.} \end{aligned}$$

2.2-4 $E(X) = (-1)\left(\dfrac{4}{9}\right) + (0)\left(\dfrac{1}{9}\right) + (1)\left(\dfrac{4}{9}\right) = 0;$

$E(X^2) = (-1)^2\left(\dfrac{4}{9}\right) + (0)^2\left(\dfrac{1}{9}\right) + (1)^2\left(\dfrac{4}{9}\right) = \dfrac{8}{9};$

$E(3X^2 - 2X + 4) = 3\left(\dfrac{8}{9}\right) - 2(0) + 4 = \dfrac{20}{3}.$

2.2-6 $E(X) = \$\,499(0.001) - \$\,1(0.999) = -\$\,0.50.$

2.2-8 Note that $\displaystyle\sum_{x=1}^{\infty} \frac{6}{\pi^2 x^2} = \frac{6}{\pi^2} \sum_{x=1}^{\infty} \frac{1}{x^2} = \frac{6}{\pi^2}\,\frac{\pi^2}{6} = 1$, so this is a p.d.f.

$$E(X) = \sum_{x=1}^{\infty} x\,\frac{6}{\pi^2 x^2} = \frac{6}{\pi^2} \sum_{x=1}^{\infty} \frac{1}{x}$$

and it is well known that the sum of this harmonic series is not finite.

2.2-10 $E(|X - c|) = \dfrac{1}{7} \displaystyle\sum_{x \in S} |x - c|$, where $S = \{1, 2, 3, 5, 15, 25, 50\}$.
When $c = 5$,

$$E(|X - 5|) = \frac{1}{7}\left[(5 - 1) + (5 - 2) + (5 - 3) + (5 - 5) + (15 - 5) + (25 - 5) + (50 - 5)\right].$$

If c is either increased or decreased by 1, this expectation is increased by 1/7. Thus $c = 5$, the median, minimizes this expectation while $b = E(X) = \mu$, the mean, minimizes $E[(X - b)^2]$. You could also let $h(c) = E(\,|\,X - c\,|\,)$ and show that $h'(c) = 0$ when $c = 5$.

2.2-12 $(1) \cdot \dfrac{15}{36} + (-1) \cdot \dfrac{21}{36} = \dfrac{-6}{36} = \dfrac{-1}{6};$

$(1) \cdot \dfrac{15}{36} + (-1) \cdot \dfrac{21}{36} = \dfrac{-6}{36} = \dfrac{-1}{6};$

$(4) \cdot \dfrac{6}{36} + (-1) \cdot \dfrac{30}{36} = \dfrac{-6}{36} = \dfrac{-1}{6}.$

2.2-14 **(a)** The average class size is $\dfrac{(16)(25) + (3)(100) + (1)(300)}{20} = 50;$

(b)

$$f(x) = \begin{cases} 0.4, & x = 25, \\ 0.3, & x = 100, \\ 0.3, & x = 300, \end{cases}$$

(c) $E(X) = 25(0.4) + 100(0.3) + 300(0.3) = 130.$

2.3 The Mean, Variance, and Standard Deviation

2.3-2 **(a)**

$$\begin{aligned}
\mu &= E(X) \\
&= \sum_{x=1}^{3} x \frac{3!}{x!\,(3-x)!}\left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{3-x} \\
&= 3\left(\frac{1}{4}\right)\sum_{k=0}^{2} \frac{2!}{k!\,(2-k)!}\left(\frac{1}{4}\right)^k \left(\frac{3}{4}\right)^{2-k} \\
&= 3\left(\frac{1}{4}\right)\left(\frac{1}{4}+\frac{3}{4}\right)^2 = \frac{3}{4}; \\
E[X(X-1)] &= \sum_{x=2}^{3} x(x-1) \frac{3!}{x!\,(3-x)!}\left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{3-x} \\
&= 2(3)\left(\frac{1}{4}\right)^2 \frac{3}{4} + 6\left(\frac{1}{4}\right)^3 \\
&= 6\left(\frac{1}{4}\right)^2 = 2\left(\frac{1}{4}\right)\left(\frac{3}{4}\right); \\
\sigma^2 &= E[X(X-1)] + E(X) - \mu^2 \\
&= (2)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right) + \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 \\
&= (2)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right) + \left(\frac{3}{4}\right)\left(\frac{1}{4}\right) = 3\left(\frac{1}{4}\right)\left(\frac{3}{4}\right);
\end{aligned}$$

(b)

$$\begin{aligned}
\mu &= E(X) \\
&= \sum_{x=1}^{4} x \frac{4!}{x!\,(4-x)!}\left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x} \\
&= 4\left(\frac{1}{2}\right)\sum_{k=0}^{3} \frac{3!}{k!\,(3-k)!}\left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{3-k} \\
&= 4\left(\frac{1}{2}\right)\left(\frac{1}{2}+\frac{1}{2}\right)^3 = 2;
\end{aligned}$$

$$\begin{aligned}
E[X(X-1)] &= \sum_{x=2}^{4} x(x-1)\,\frac{4!}{x!\,(4-x)!}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{4-x} \\
&= 2(6)\left(\frac{1}{2}\right)^{4} + (6)(4)\left(\frac{1}{2}\right)^{4} + (12)\left(\frac{1}{2}\right)^{4} \\
&= 48\left(\frac{1}{2}\right)^{4} = 12\left(\frac{1}{2}\right)^{2}; \\
\sigma^2 &= (12)\left(\frac{1}{2}\right)^{2} + \frac{4}{2} - \left(\frac{4}{2}\right)^{2} = 1.
\end{aligned}$$

2.3-4 $E[(X-\mu)/\sigma] = (1/\sigma)[E(X)-\mu] = (1/\sigma)(\mu-\mu) = 0;$

$$E\{[(X-\mu)/\sigma]^2\} = (1/\sigma^2)E[(X-\mu)^2] = (1/\sigma^2)(\sigma^2) = 1.$$

2.3-6 **(a)** $f(x) = P(X=x) = \dfrac{\dbinom{6}{x}\dbinom{43}{6-x}}{\dbinom{49}{6}}, \qquad x = 0,1,2,3,4,5,6;$

(b)

$$\begin{aligned}
\mu_X &= \sum_{x=0}^{6} x f(x) = \frac{36}{49} = 0.7347, \\
\sigma_X^2 &= \sum_{x=0}^{6} (x-\mu)^2 f(x) = \frac{5{,}547}{9{,}604} = 0.5776; \\
\sigma_X &= \frac{43}{98}\sqrt{3} = 0.7600;
\end{aligned}$$

(c) $f(0) = \dfrac{435{,}461}{998{,}844} > \dfrac{412{,}542}{998{,}844} = f(1);\quad X = 0$ is most likely to occur.

(d) The numbers are reasonable because

$$\begin{aligned}
(25{,}000{,}000)f(6) &= 1.79; \\
(25{,}000{,}000)f(5) &= 461.25; \\
(25{,}000{,}000)f(4) &= 24{,}215.49;
\end{aligned}$$

(e) The respective expected values, $(138)f(x)$, for $x = 0,1,2,3$, are 60.16, 57.00, 18.27, and 2.44, so the results are reasonable. See Figure 2.3-6 for a comparison of the theoretical probability histogram and the histogram of the data.

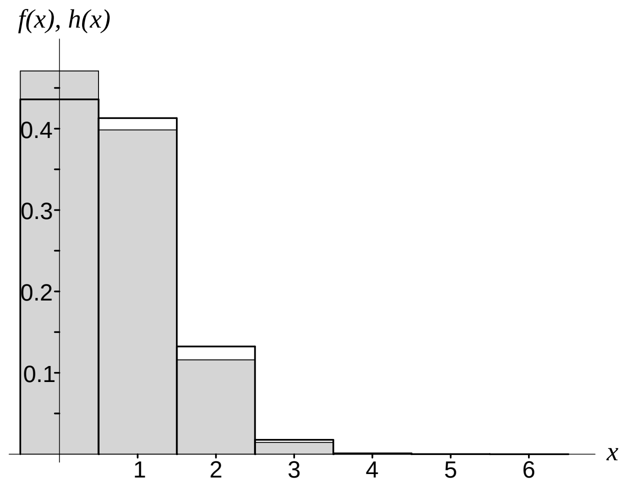

Figure 2.3–6: Empirical (shaded) and theoretical histograms for LOTTO

2.3-8 **(a)** Out of the 75 numbers, first select $x-1$ of which 23 are selected out of the 24 good numbers on your card and the remaining $x-1-23$ are selected out of the 51 bad numbers. There is now one good number to be selected out of the remaining $75-(x-1)$.

(b) The mode is 75.

(c) $\mu = \dfrac{1824}{25} = 72.96.$

(d) $E[X(X+1)] = \dfrac{70,224}{13} = 5,401.846154.$

(e) $\sigma^2 = \dfrac{46,512}{8,125} = 5.724554;\ \ \sigma = 2.3926.$

(f) **(i)** $\overline{x} = 72.78$, **(ii)** $s^2 = 8.7187879$, **(iii)** $s = 2.9528$, **(iv)** 5378.34.

(g)

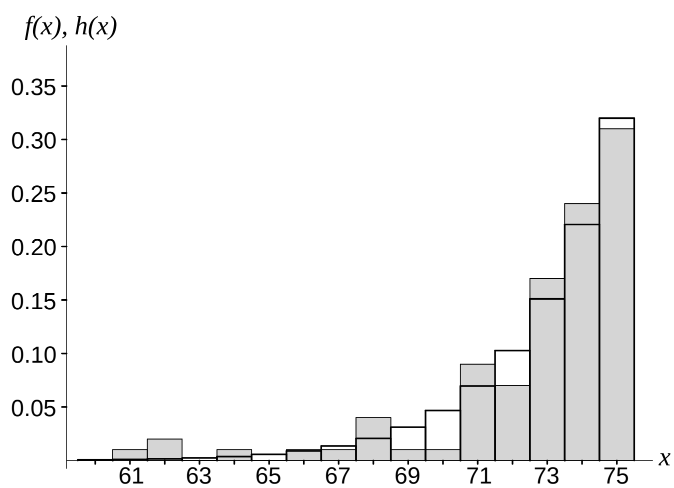

Figure 2.3–8: Bingo "cover-up" comparisons

2.3-10 **(a)** $P(X \ge 1) = \dfrac{2^1}{\binom{3}{1}} = \dfrac{2}{3};$

(b) $\sum_{k=1}^{5} P(X \geq k) = P(X = 1) + 2P(X = 2) + \cdots + 5P(X = 5) = \mu$;

(c) $\mu = \dfrac{5,168}{3,465} = 1.49149$;

(d) In the limit, $\mu = \dfrac{\pi}{2}$.

2.3-12 $\overline{x} = \dfrac{409}{50} = 8.18$.

2.3-14 $f(1) = \dfrac{3}{8}$, $f(2) = \dfrac{2}{8}$, $f(3) = \dfrac{3}{8}$

$\mu = 1 \cdot \dfrac{3}{8} + 2 \cdot \dfrac{2}{8} + 3 \cdot \dfrac{3}{8} = 2$,

$\sigma^2 = 1^2 \cdot \dfrac{3}{8} + 2^2 \cdot \dfrac{2}{8} + 3^2 \cdot \dfrac{3}{8} - 2^2 = \dfrac{3}{4}$.

2.3-16 **(a)** $\overline{x} = \dfrac{4}{3} = 1.333$;

(b) $s^2 = \dfrac{88}{69} = 1.275$.

2.3-18 **(a)** $[3, 19, 16, 9]$;

(b) $\overline{x} = \dfrac{125}{47} = 2.66, s = 0.87$;

(c)

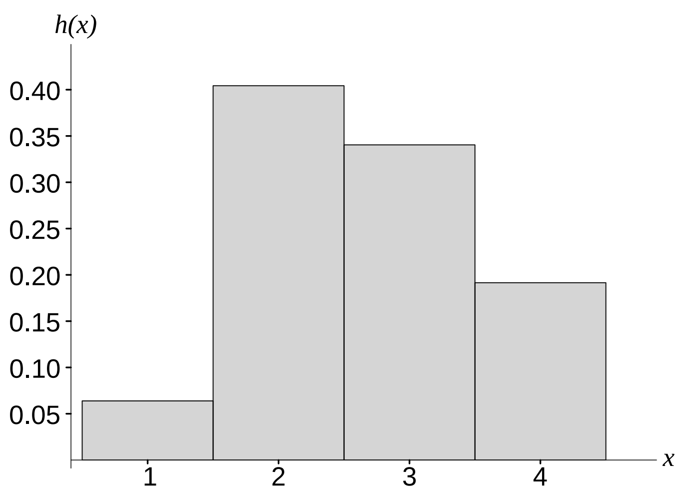

Figure 2.3–18: Number of pets

2.4 Bernoulli Trials and the Binomial Distribution

2.4-2 $f(-1) = \dfrac{11}{18}, f(1) = \dfrac{7}{18}$;

$\mu = (-1)\dfrac{11}{18} + (1)\dfrac{7}{18} = -\dfrac{4}{18}$;

$\sigma^2 = \left(-1 + \dfrac{4}{18}\right)^2 \left(\dfrac{11}{18}\right) + \left(1 + \dfrac{4}{18}\right)^2 \left(\dfrac{7}{18}\right) = \dfrac{77}{81}$.

2.4-4 **(a)** $P(X \le 5) = 0.5269$;

(b) $P(X \ge 6) = 1 - P(X \le 5) = 0.4731$;

(c) $P(X \le 7) - P(X \le 6) = 0.8883 - 0.7393 = 0.1490$;

(d) $\mu = (12)(0.45) = 5.4,\ \sigma^2 = (12)(0.45)(0.55) = 2.97, \quad \sigma = \sqrt{2.97} = 1.723.$

2.4-6 **(a)** X is $b(7,\ 0.15)$;

(b)
$$\begin{aligned} \text{(i)}\quad P(X \ge 2) &= 1 - P(X \le 1) = 1 - 0.7166 = 0.2834; \\ \text{(ii)}\quad P(X = 1) &= P(X \le 1) - P(X \le 0) = 0.7166 - 0.3206 = 0.3960; \\ \text{(iii)}\quad P(X \le 3) &= 0.9879. \end{aligned}$$

2.4-8 **(a)** X is $b(15, 0.2)$,

(b) $\mu = 15(0.2) = 3, \quad \sigma^2 = 15(0.2)(0.8) = 2.4, \quad \sigma = \sqrt{2.4} = 1.549$;

(c) $P(X \ge 5) = 1 - P(X \le 4) = 1 - 0.8358 = 0.1642$.

2.4-10 **(a)** X is $b(6,\ 0.05)$;

(b) $\mu = 6(0.05) = 0.3; \quad \sigma^2 = 6(0.05)(0.95) = 0.285$;

(c)
$$\begin{aligned} \textbf{(i)}\quad P(X = 0) &= 0.7351; \\ \textbf{(ii)}\quad P(X \le 1) &= 0.9672; \\ \textbf{(iii)}\quad P(X \ge 2) &= 1 - P(X \le 1) = 0.0328. \end{aligned}$$

2.4-12 **(a)** $\mu = 14(0.55) = 7.7,\ \sigma^2 = 14(0.55)(0.45) = 3.465$;

(b)
$$\begin{aligned} P(X < 8) &= P(X \le 7) = P(14 - X \ge 14 - 7) \\ &= P(14 - X \ge 7) = 1 - 0.5461 = 0.4539, \\ P(X > 6) &= P(14 - X < 14 - 6) \\ &= P(14 - X \le 7) = 0.7414. \end{aligned}$$

2.4-14 **(a)** X is $b(8,\ 0.90)$;

(b)
$$\begin{aligned} \textbf{(i)}\quad P(X = 8) &= P(8 - X = 0) = 0.4305; \\ \textbf{(ii)}\quad P(X \le 6) &= P(8 - X \ge 2) \\ &= 1 - P(8 - X \le 1) = 1 - 0.8131 = 0.1869; \\ \textbf{(iii)}\quad P(X \ge 6) &= P(8 - X \le 2) = 0.9619. \end{aligned}$$

2.4-16 **(a)**
$$f(x) = \begin{cases} 125/216, & x = -1, \\ 75/216, & x = 1, \\ 15/216, & x = 2, \\ 1/216, & x = 3; \end{cases}$$

(b)
$$\begin{aligned} \mu &= (-1) \cdot \frac{125}{216} + (1) \cdot \frac{75}{216} + (2) \cdot \frac{15}{216} + (3) \cdot \frac{1}{216} = -\frac{17}{216}; \\ \sigma^2 &= E(X^2) - \mu^2 = \frac{269}{216} - \left(-\frac{17}{216}\right)^2 = 1.2392; \\ \sigma &= 1.11; \end{aligned}$$

(c) See Figure 2.4-16.

(d)
$$\begin{aligned}\overline{x} &= \frac{-1}{100} = -0.01;\\ s^2 &= \frac{100(129)-(-1)^2}{100(99)} = 1.3029;\\ s &= 1.14.\end{aligned}$$

(e)

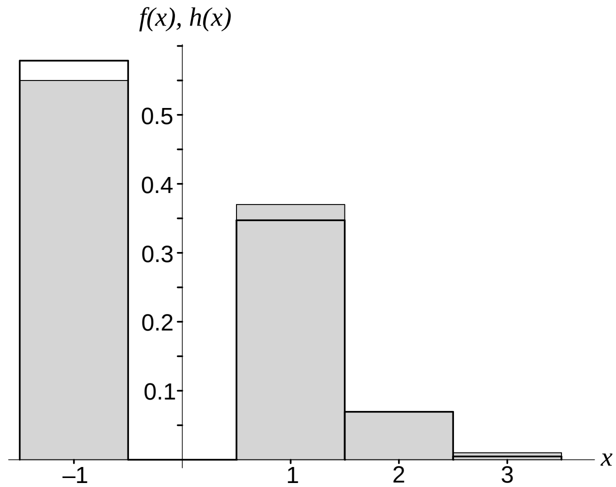

Figure 2.4–16: Losses in chuck-a-luck

2.4-18 Let X equal the number of winning tickets when n tickets are purchased. Then

$$\begin{aligned}P(X \geq 1) &= 1 - P(X=0)\\ &= 1 - \left(\frac{9}{10}\right)^n.\end{aligned}$$

(a)
$$\begin{aligned}1-(0.9)^n &= 0.50\\ (0.9)^n &= 0.50\\ n\ln 0.9 &= \ln 0.5\\ n &= \frac{\ln 0.5}{\ln 0.9} = 6.58\end{aligned}$$

so $n = 7$.

(b)
$$\begin{aligned}1-(0.9)^n &= 0.95\\ (0.9)^n &= 0.05\\ n &= \frac{\ln 0.05}{\ln 0.09} = 28.43\end{aligned}$$

so $n = 29$.

2.4-20 $\dfrac{(0.1)(1-0.95^5)}{(0.4)(1-0.97^5)+(0.5)(1-0.98^5)+(0.1)(1-0.95^5)} = 0.178.$

2.4-22 **(a)** $1-0.01^4 = 0.99999999$; **(b)** $0.99^4 = 0.960596$.

2.5 The Moment-Generating Function

2.5-2 **(a)** **(i)** $b(5,\ 0.7)$; **(ii)** $\mu = 3.5, \sigma^2 = 1.05$; **(iii)** 0.1607;

(b) **(i)** geometric, $p = 0.3$; **(ii)** $\mu = 10/3, \sigma^2 = 70/9$; **(iii)** 0.51;

(c) **(i)** Bernoulli, $p = 0.55$; **(ii)** $\mu = 0.55, \sigma^2 = 0.2475$; **(iii)** 0.55;

(d) **(ii)** $\mu = 2.1, \sigma^2 = 0.89$; **(iii)** 0.7;

(e) **(i)** negative binomial, $p = 0.6, r = 2$; **(ii)** $10/3$, $\sigma^2 = 20/9$; **(iii)** 0.36;

(f) **(i)** discrete uniform on $1, 2, \ldots, 10$; **(ii)** 5.5, 8.25; **(iii)** 0.2.

2.5-4 **(a)** $f(x) = \left(\dfrac{364}{365}\right)^{x-1}\left(\dfrac{1}{365}\right), \qquad x = 1, 2, 3, \ldots,$

(b)
$$\begin{aligned}\mu &= \frac{1}{\dfrac{1}{365}} = 365,\\ \sigma^2 &= \frac{\dfrac{364}{365}}{\left(\dfrac{1}{365}\right)^2} = 132{,}860,\\ \sigma &= 364.500;\end{aligned}$$

(c)
$$\begin{aligned}P(X > 400) &= \left(\frac{364}{365}\right)^{400} = 0.3337,\\ P(X < 300) &= 1 - \left(\frac{364}{365}\right)^{299} = 0.5597.\end{aligned}$$

2.5-6 $P(X \ge 100) = P(X > 99) = (0.99)^{99} = 0.3697.$

2.5-8 $\dbinom{10-1}{5-1}\left(\dfrac{1}{2}\right)^5\left(\dfrac{1}{2}\right)^5 = \dfrac{126}{1024} = \dfrac{63}{512}.$

2.5-10 **(a)** Negative binomial with $r = 10, p = 0.6$ so

$$\mu = \frac{10}{0.60} = 16.667,\ \sigma^2 = \frac{10(0.40)}{(0.60)^2} = 11.111,\ \sigma = 3.333;$$

(b) $P(X = 16) = \dbinom{15}{9}(0.60)^{10}(0.40)^6 = 0.1240.$

2.5-12
$$\begin{aligned}P(X > k + j \,|\, X > k) &= \frac{P(X > k + j)}{P(X > k)}\\ &= \frac{q^{k+j}}{q^k} = q^j = P(X > j).\end{aligned}$$

2.5-14 **(b)**
$$\begin{aligned}\sum_{x=2}^{\infty} f(x) &= \sum_{x=2}^{\infty} \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{x-1} - \left(\frac{1-\sqrt{5}}{2}\right)^{x-1}\right]\left(\frac{1}{2^x}\right)\\ &= \frac{2}{\sqrt{5}(1+\sqrt{5})}\sum_{x=2}^{\infty}\frac{(1+\sqrt{5})^x}{4^x} - \frac{2}{\sqrt{5}(1-\sqrt{5})}\sum_{x=2}^{\infty}\frac{(1-\sqrt{5})^x}{4^x}\\ &= \text{(fill in missing steps)}\\ &= 1;\end{aligned}$$

$$\textbf{(c)}\quad E(X) = \sum_{x=2}^{\infty} \frac{x}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{x-1} - \left(\frac{1-\sqrt{5}}{2}\right)^{x-1}\right]\left(\frac{1}{2^x}\right)$$

$$= \frac{1}{2\sqrt{5}}\sum_{x=1}^{\infty}\left[x\left(\frac{1+\sqrt{5}}{4}\right)^{x-1} - x\left(\frac{1-\sqrt{5}}{4}\right)^{x-1}\right]$$

$$= \frac{1}{2\sqrt{5}}\left[\frac{1}{(1-(1+\sqrt{5})/4)^2} - \frac{1}{(1-(1-\sqrt{5}/4)^2}\right]$$

$$= \text{(fill in missing steps)}$$

$$= 6;$$

$$\textbf{(d)}\quad E[X(X-1)] = \sum_{x=2}^{\infty} x(x-1)\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{x-1} - \left(\frac{1-\sqrt{5}}{2}\right)^{x-1}\right]\left(\frac{1}{2^x}\right)$$

$$= \frac{1}{2\sqrt{5}}\sum_{x=2}^{\infty} x(x-1)\left[\left(\frac{1+\sqrt{5}}{4}\right)^{x-1} - \left(\frac{1-\sqrt{5}}{4}\right)^{x-1}\right]$$

$$= \frac{1}{2\sqrt{5}}\left[\frac{1+\sqrt{5}}{4}\sum_{x=2}^{\infty} x(x-1)\left(\frac{1+\sqrt{5}}{4}\right)^{x-2} - \frac{1-\sqrt{5}}{4}\sum_{x=2}^{\infty} x(x-1)\left(\frac{1-\sqrt{5}}{4}\right)^{x-2}\right]$$

$$= \frac{1}{2\sqrt{5}}\left[\frac{2\left(\frac{1+\sqrt{5}}{4}\right)}{\left(1-\frac{1+\sqrt{5}}{4}\right)^3} - \frac{2\left(\frac{1-\sqrt{5}}{4}\right)}{\left(1-\frac{1-\sqrt{5}}{4}\right)^3}\right]$$

$$= \text{(fill in missing steps)}$$

$$= 52;$$

$$\sigma^2 = E[X(X-1)] + E(X) - \mu^2 = 52 + 6 - 36 = 22;$$

$$\sigma = \sqrt{22} = 4.690.$$

2.5-16 **(a)** $1/(1/6) = 6$;

(b) $1-(5/6)^2 = 11/36, \quad 1/(11/36) = 36/11$;

(c)

$$\frac{1}{1-(5/6)^n} \le 2,$$
$$0.5 \le 1-(5/6)^n,$$
$$(5/6)^n \le 0.5,$$
$$n \ge 4.$$

2.5-18 $M(t) = 1 + \frac{5t}{1!} + \frac{5t^2}{2!} + \frac{5t^3}{3!} + \cdots = e^{5t}$,

$f(x) = 1, \qquad x = 5.$

2.5-20 **(a)**

$$\begin{aligned} R(t) &= \ln(1-p+pe^t), \\ R'(t) &= \left[\frac{pe^t}{1-p+pe^t}\right]_{t=0} = p, \\ R''(t) &= \left[\frac{(1-p+pe^t)(pe^t)-(pe^t)(pe^t)}{(1-p+pe^t)^2}\right]_{t=0} = p(1-p); \end{aligned}$$

(b)

$$\begin{aligned} R(t) &= n\ln(1-p+pe^t), \\ R'(t) &= \left[\frac{npe^t}{1-p+pe^t}\right]_{t=0} = np, \\ R''(t) &= n\left[\frac{(1-p+pe^t)(pe^t)-(pe^t)(pe^t)}{(1-p+pe^t)^2}\right]_{t=0} = np(1-p); \end{aligned}$$

(c)

$$\begin{aligned} R(t) &= \ln p + t - \ln[1-(1-p)e^t], \\ R'(t) &= \left[1+\frac{(1-p)e^t}{1-(1-p)e^t}\right]_{t=0} = 1+\frac{1-p}{p} = \frac{1}{p}, \\ R''(t) &= \left[(-1)\{1-(1-p)e^t\}^2\{-(1-p)e^t\}\right]_{t=0} = \frac{1-p}{p}; \end{aligned}$$

(d)

$$\begin{aligned} R(t) &= r\left[\ln p + t - \ln\{1-(1-p)e^t\}\right], \\ R'(t) &= r\left[\frac{1}{1-(1-p)e^t}\right]_{t=0} = \frac{r}{p}, \\ R''(t) &= r\left[(-1)\{1-(1-p)e^t\}^{-2}\{-(1-p)e^t\}\right]_{t=0} = \frac{r(1-p)}{p^2}. \end{aligned}$$

2.5-22 $(0.7)(0.7)(0.3) = 0.147$.

2.5-24 **(a)** $0.9^{12} = 0.282$. Note that "miss" = "success";

(b) $\binom{29}{2}(0.9)^{27}(0.1)^2(0.1) = 0.0236$.

2.6 The Poisson Distribution

2.6-2 $\lambda = \mu = \sigma^2 = 3$ so $P(X=2) = 0.423 - 0.199 = 0.224$.

2.6-4

$$\begin{aligned} 3\,\frac{\lambda^1 e^{-\lambda}}{1!} &= \frac{\lambda^2 e^{-\lambda}}{2!} \\ e^{-\lambda}\lambda(\lambda-6) &= 0 \\ \lambda &= 6 \end{aligned}$$

Thus $P(X=4) = 0.285 - 0.151 = 0.134$.

2.6-6 $\lambda = (1)(50/100) = 0.5$, so $P(X=0) = e^{-0.5}/0! = 0.607$.

2.6-8 $np = 1000(0.005) = 5$;

(a) $P(X \le 1) \approx 0.040$;

(b) $P(X = 4,5,6) = P(X \le 6) - P(X \le 3) \approx 0.762 - 0.265 = 0.497$.

2.6-10 $\sigma = \sqrt{9} = 3$,

$$P(3 < X < 15) = P(X \le 14) - P(X \le 3) = 0.959 - 0.021 = 0.938.$$

2.6-12 **(a)** $[17, 47, 63, 63, 49, 28, 21, 11, 1]$;

(b) $\overline{x} = 303/100 = 3.03$, $s^2 = 4,141/1,300 = 3.193$, yes;

(c)

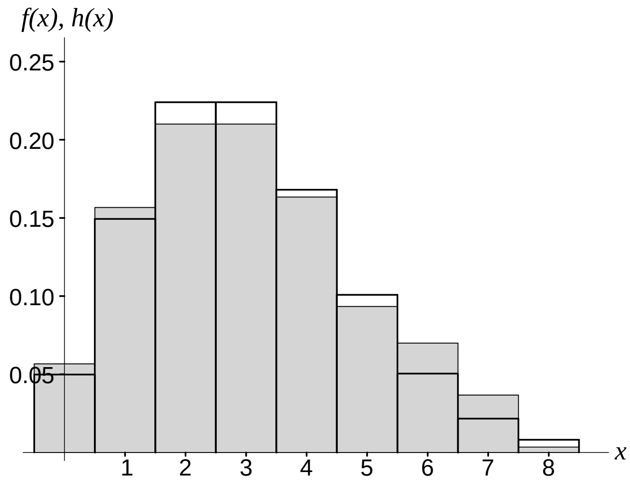

Figure 2.6–12: Background radiation

(d) The fit is very good and the Poisson distribution seems to provide an excellent probability model.

2.6-14 **(a)**

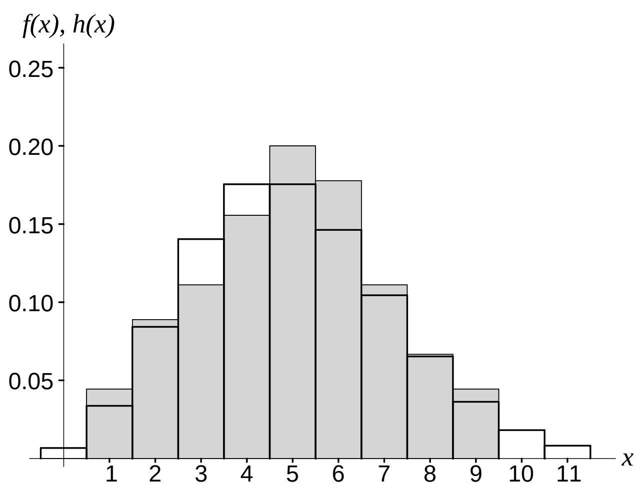

Figure 2.6–14: Green peanut m&m's

(b) The fit is quite good. Also $\overline{x} = 4.956$ and $s^2 = 4.134$ are close to each other.

2.6-16 **(a)**

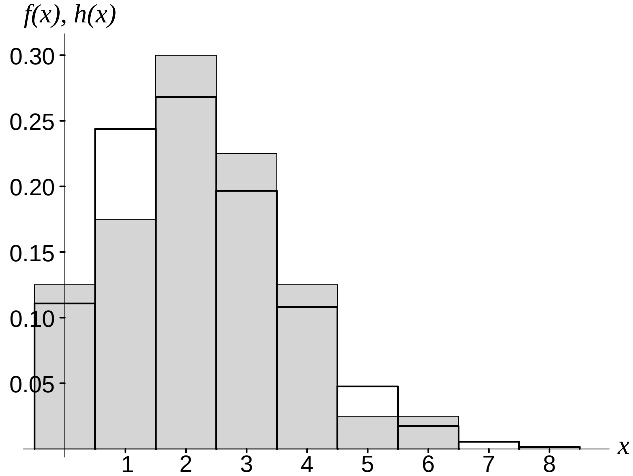

Figure 2.6–16: Bad records on a computer tape

(b) Yes. Again note that $\overline{x} = 2.225$ and $s^2 = 2.025$ are close to each other.

2.6-18

$$\begin{aligned} OC(p) &= P(X \le 3) \approx \sum_{x=0}^{3} \frac{(400p)^x e^{-400p}}{x!}; \\ OC(0.002) &\approx 0.991; \\ OC(0.004) &\approx 0.921; \\ OC(0.006) &\approx 0.779; \\ OC(0.01) &\approx 0.433; \\ OC(0.02) &\approx 0.042. \end{aligned}$$

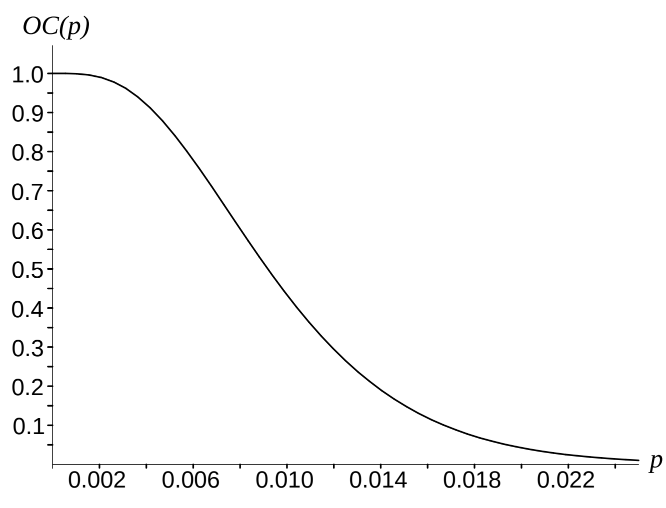

Figure 2.6–18: Operating characteristic curve

2.6-20 Since $E(X) = 0.2$, the expected loss is $(0.02)(\$10,000) = \$2,000$.

2.6-22 Use the Poisson approximation. If $n = 200$ and $p = 0.01$, then $\lambda = 2$. Using Table III, $P(X \le 5) = 0.983$.

If $n = 200$ and $p = 0.05$, then $\lambda = 10$. Using Table III, $P(X \le 5) = 0.067$.

2.6-24 Using Minitab,

(a) $P(X = 0) = 0.0821$;

(b) $P(X = 3) = 0.2138$;

(c) $P(X \geq 3) = 1 - 0.5438 = 0.4562$;

(d) $P(X \leq 3) = 0.7576$.